solution of problems of noncircular cylindrical shell vibrations with low variability; such vibrations require special analysis.

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GENERALIZED CYCLIC DISPLACEMENTS

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We consider the generalized cyclic displacements of holonomic mechanical systems with a finite number of degrees of freedom, and their application to integration of the equations of motion.

N. G. Chetaev in [1] turned his attention to the formulation of problems dealing with general properties of mechanical systems and connected with the groups of transformations which leave the basic mechanical functions invariant. It was he who introduced [2] the concept of cyclic displacement of a mechanical system with smooth holonomic constraints. This concept was enlarged in [3] in the course of considering a particular case of motion of a mechanical system with three degrees of freedom.

1. Let us consider a mechanical system with smooth holonomic constraints, and with k degrees of freedom. We assume that the position of the system is determined by the real dependent variables $x_1, x_2, \ldots, x_n$ $(n > k)$. The possible displacements of this system are determined by an intransitive, k-membered group of infinitesimal operators

$$X_\alpha = \sum_{i=1}^{n} \xi_\alpha^{\ i} \frac{\partial}{\partial x_i} \qquad (\alpha = 1, 2, \ldots, k)$$

The problem of constructing the groups of possible displacements was studied in [4].

The variations of the function $f(t, x_1, \ldots, x_n)$ over the possible (δf) and the real (df) displacements of the system are, respectively,

$$\delta f = \sum_{\alpha=1}^{k} \omega_\alpha X_\alpha f, \quad df = \left(\frac{\partial f}{\partial t} + \sum_{\alpha=1}^{k} \eta_\alpha X_\alpha f\right) dt \tag{1.1}$$

where ω_α and η_α are the mutually independent parameters of the possible and the real displacements, respectively.

The possible displacements X_α satisfying the conditions

$$X_\alpha (L) = 0 \qquad (X_\alpha, X_\beta) = 0$$

were defined by Chetaev in [2] as the cyclic displacements. An example of such displacements was investigated in [5]. Replacing the Poincaré parameters by the variables

$$y_\alpha = \partial T / \partial \eta_\alpha \tag{1.2}$$

Chetaev established the canonical equations and a partial differential equation for the action function V, in the form:

$$\frac{dy_s}{dt} = \sum c_{\alpha s \beta} \eta_\alpha y_\beta - X_s (H), \quad \eta_s = \frac{\partial H}{\partial y_s} \quad (s = 1, \ldots, k) \tag{1.3}$$

$$\frac{\partial V}{\partial t} + H(t, x_1, \ldots x_n, X_1 V, \ldots, X_k V) = 0 \tag{1.4}$$

$$H = \sum_{i=1}^{k} \eta_i y_i - L = H(t, x_1, \ldots, x_n, y_1, \ldots, y_k)$$

$$V = V(t, x_1, \ldots, x_n, x_1^\circ, \ldots, x_n^\circ)$$

Here H is the Hamiltonian, $L = T + U$ is the Largangian and U is the force function of the system.

2. The velocities x_i' are linear functions of the variables $\eta_1, \ldots, \eta_k$, therefore the kinetic energy T can be written in the form

$$T = T_2 + T_1 + T_0, \quad T_2 = \Sigma g_{\alpha\beta} \eta_\alpha \eta_\beta, \quad T_1 = \Sigma a_\alpha \eta_\alpha$$

Since T_2 is real, we can assume without loss of generality that it is specified in symmetric form, i.e. $g_{\alpha\beta} = g_{\beta\alpha}$. The quantities T_0 and U are independent of η_α.

Let us introduce the generalized cyclic displacements X_s $(s = r, \ldots, k;\ r < k)$, satisfying the following conditions:

1) when $g_{ij} = \delta_{ij} g_i$ $(i, j = 1, \ldots, k)$, the kinetic energy of the system can be reduced to the above form by means of the known transformations;

2) $(X_s, X_i) = 0$

3) $X_s (\partial L / d\eta_i) = 0$

4) $X_i X_s (U) = 0 \qquad (i = 1, \ldots, k;\ i \neq s)$

For simplicity, let us consider the case when $\partial L / \partial t = 0$. Using the formulas (1.2) we introduce the system of canonical variables $x_1, \ldots, x_n, y_1, \ldots, y_k$ and the Hamiltonian H. The total integral of the partial differential equation (1.4) is, according to the known substitution of Imshenetskii, equal to the sum $V = -ht + W$, where h is the constant term of the energy integral and W is the total integral of the equation

$$H(x_1, \ldots, x_n, X_1 W, \ldots, X_k W) = h \tag{2.1}$$

The above equation is obtained from the general Jacobi energy integral in which the variables y_i $(i = 1, \ldots, k)$, replace the variables $\eta_1, \ldots, \eta_k$ through the system (1.2), and y_i are in turn replaced by the expressions for $X_i(W)$. We shall denote these transformations by an asterisk accompanying the function T_2. Thus, the function H in (2.1) is

$$H = T_2{}^* - T_0 - U$$

$$T_2{}^* = T_2{}^*(x_1, \ldots, x_n, X_1 W, \ldots, X_k W) = \frac{1}{2} \sum_{i=1}^{k} \frac{1}{g_i} (X_i W)^2$$

In accordance with the canonical equations (1.3) the variations of the functions $T_0 + U$ and W over the real displacements can be expressed, by virtue of (1.1), in the canonical system of variables in the form

$$d(T_0 + U) = \left\{ \sum_{\alpha=1}^{k} \frac{\partial H}{\partial y_\alpha} X_\alpha (T_0 + U) \right\} dt \tag{2.2}$$

$$dW = \left\{ \sum_{\alpha=1}^{k} \frac{\partial H}{\partial y_\alpha} X_\alpha (W) \right\} dt \tag{2.3}$$

Using (2.2) we can write (2.1) in the form

$$\frac{1}{2} \sum_{i=1}^{k} \frac{1}{g_i} (X_i W)^2 - \int \left\{ \sum_{\alpha=1}^{k} \frac{\partial H}{\partial y_\alpha} X_\alpha (T_0 + U) \right\} dt = h \tag{2.4}$$

By virtue of the condition (1), in the canonical system of variables, condition (3) is equivalent to the condition

$$X_s (\partial H / \partial y_i) = 0 \qquad (i = 1, \ldots, k;\ s = r, \ldots, k;\ s \neq i)$$

The latter, together with the condition (4), enables us to obtain the following expression from (2.2)

$$X_s [d(T_0 + U)] = X_s \left[\frac{\partial H}{\partial y_s} X_s (T_0 + U) \right] dt$$

This makes it possible to separate a part of variables in (2.4). Let us assume that

$$W = \sum_{s=r}^{k} W_s + W_0 + \text{const}$$

$$X_\alpha (W_s) = 0 \qquad (\alpha = 1, \ldots, k;\ s = r, \ldots, k;\ s \neq \alpha)$$

then by (2.3) we have

$$dW_s = \frac{\partial H}{\partial y_s} X_s (W_s)\, dt, \quad dW_0 = \left\{ \sum_{\alpha=1}^{k} \frac{\partial H}{\partial y_\alpha} X_\alpha (W_0) \right\} dt \ (s = r, \ldots, k) \tag{2.5}$$

$$dW = \sum_{s=r}^{k} dW_s + dW_0 = \left\{ \sum_{s=r}^{k} \frac{\partial H}{\partial y_s} X_s (W_s) + \sum_{\alpha=1}^{k} \frac{\partial H}{\partial y_\alpha} X_\alpha (W_0) \right\} dt$$

We introduce a substitution for (2.4) analogous to the Imshenetskii substitution in the following manner: all terms of the equation not vanishing under the displacement X_s are assumed equal to constants l_s $(s = r, \ldots, k)$

$$\frac{1}{2g_s}\left[X_s(W_s)\right]^2 - \int \frac{\partial H}{\partial y_s} X_s(T_0 + U)\,dt = l_s$$

By (2.5) we have

$$W_s = \int \frac{\partial H}{\partial y_s}\left[2g_s \int \frac{\partial H}{\partial y_s} X_s(T_0 + U)\,dt + 2g_s l_s\right]^{1/2} dt$$

The function W_0 satisfies the following partial differential equation:

$$T_2^*(x_1, \ldots, x_n, X_1 W_0, \ldots, X_{r-1} W_0) - \int\left\{\sum_{\nu=1}^{r-1} \frac{\partial H}{\partial y_\nu} X_\nu (T_0 + U)\right\} dt = h - \sum_{s=r}^{k} l_s \qquad (2.6)$$

If the variables are completely separable, i.e. $r = 1$, then the solution of the problem is obtained from the Chetaev theorem

$$X_\alpha(V) = y_\alpha, \qquad X_\alpha^\circ(V) = -y_\alpha^\circ \qquad (\alpha = 1, \ldots k)$$

where the last group of equations gives the law of motion in its implicit form.

An example of generalized cyclic displacements is discussed in [3].

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